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Existence of Solutions for Three-order M-point Boundary Value Problems at Resonance

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Abstract: In this note, the theory of coincidence degree is used to study the existence of solutions for Three-order m-point boundary value problems at resonance. Under the condition of KerL=3, some relevant results in the literature are improved.

1. Introduction

In recent years, the multi-point boundary value problem of differential equations has been extensively and deeply studied, and some results have been obtained [1-4]. However, most of the existing research methods focus on the principle of compression mapping and the fixed point theorem. The theory of coincidence degree studies the boundary value problem, especially in the case of resonance, the results of studying the non-local boundary value problem are few. Therefore, based on the above literature, this paper mainly studies the following three types of third-order ordinary differential equation m-point boundary value Resonance problem:

$$x''' = f(t, x(t), x'(t), x''(t)) + e(t), \quad t\in[0,1],$$

$$x(0)=0, \quad x'(1)=\sum_{i=1}^{m} a_i x'(\varepsilon_i), \quad x(1)=\int_0^1 A(t)x(t)dt,$$

Where, $0<\varepsilon_1<\varepsilon_2<\cdots<\varepsilon_m<1$, $a_i\in R$, $i=1,2,\cdots,m$, $e\in L^1[0,1]$, $\sum_{i=1}^{m} a_i=\sum_{i=1}^{m} a_i\varepsilon_i=1$,

$f(t,x,y,z):[0,1]\times R^3\to R$ meets *Caratheodory*.

2. Background

In this paper X,Z is *Banach*z one, $L: N(L)\subset X\to Z$ zero-index Fredholm operators, $P: X\to X$, Q: $Z\to Z$ is the projection operator, $\operatorname{Im}P=KerL, KerQ=\operatorname{Im}L$, $P: X\to X$, Q: $Z\to Z$ Is a projection operator, making $\operatorname{Im}P=KerL, KerQ=\operatorname{Im}L$,

$X=KerL\oplus KerP$, $X=\operatorname{Im}L\oplus\operatorname{Im}Q$, $L|_{domL\cap KerP}: domL\cap KerP\to \operatorname{Im}L$ Reversible,

Its inverse mapping is K_p,if Ω is X Bounded open subset, $domL\cap\overline{\Omega}\neq 0$, ,if $QN(\overline{\Omega})$ has boundary, $K_p(I-Q)N:\overline{\Omega}\to X$ Tight,then $N: X\to Y$ Ω is L tight.

$C[0,1], C^2[0,1], L^1[0,1]$ Is the norm $\|x\|_\infty = max\{x: x(t), t\in[0,1]\}$, $\|x\|=\|x\|_\infty+\|x'\|_\infty+\|x''\|_\infty$, $\|x\|_1=\int_0^1|x(s)|ds$ Banach space. This article also uses Soloblev space $W^{3,1}(0,1)=\{x:[0,1]\to R \,|\, x, x', x''\in AC[0,1], x'''\in L^1[0,1]\}$.

The main tool of this paper is the following theory of coincidence degree.

Theorem A [5] L is the Fredholm operator with zero index, and is L-tight, assuming the following conditions are true:

(1) $Lx\neq\lambda Nx$, $(x,\lambda)\in[(domL\setminus KerL)\cap\partial\Omega]\times(0,1)$;

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(2) $Nx \notin \operatorname{Im} L$, $x \in KerL \cap \partial\Omega$;

(3) $\deg\{JQN, \partial\Omega \cap KerL, 0\} \neq 0$, where $Q : Z \to Z$ Is a projection calculation, making

$\operatorname{Im} L = KerQ$, $J : \operatorname{Im} Q \to KerL$;

Operator equation $Lx = Nx$ $domL \cap \overline{\Omega}$.

Note 2.1 This paper assumes that the following conditions are true.

(C1): $\int_0^1 tA(t)dt = \int_0^1 t^2 A(t)dt = 1$;

(C2): $M = \begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} \neq 0.$

$m_{11} = 2(1 - \sum_{i=1}^{m} a_i \varepsilon_i^3), m_{12} = 1 - \sum_{i=1}^{m} a_i \varepsilon_i^4,$

$m_{21} = 5(1 - \int_0^1 t^4 A(t)dt),\ m_{22} = 2(1 - \int_0^1 t^5 A(t)dt).$

3. Conclusion

$X = C^2[0,1], Z = L^1[0,1],$ Define linear operator L to satisfy $D(L) \subset X \to Z,$ Where

$$D(L) = \left\{ x \in W^{3,1}(0,1) : x(0) = 0,\ x'(1) = \sum_{i=1}^{m} a_i x'(\varepsilon_i), \int_0^1 A(t)x(t)dt \right\}$$

$Lx = x''', x \in D(L).$

Defining nonlinear operators $N : X \to Z,\ Nx(t) = f(t, x(t), x'(t), x''(t)) + e(t),\ t \in [0,1],$

Then the ordinary boundary equation resonance boundary value problem (1.1)-(1.2) is equivalent to the operator equation

Lemma 3.1 If the assumption is true, then the operator of the zero indicator.

Defining projection operator

$$Qy = (T_1 y(t))t + (T_1 y(t))t^2, \tag{1}$$

Where: $T_1 y(t) = \frac{12}{M}(M_{11}Q_1 y + M_{12}Q_2 y),\ T_2 y(t) = \frac{60}{M}(M_{21}Q_1 y + M_{22}Q_2 y),$

$Q_1 y = \int_0^1 (1-s)y(s)ds - \sum_{i=1}^{m} \int_0^{\varepsilon_i} (\varepsilon_i - s)y(s)ds,$

$Q_2 y = \int_0^1 (1-s)^2 y(s)ds - \int_0^1 A(t) \int_0^t (t-s)^2 y(s)dsdt,$

$M_{11} = m_{22}, M_{12} = -m_{21}, M_{21} = -m_{12}, M_{22} = m_{11}.$

KerL$= \{x \in D(L) : x(t) = at + bt^2, a, b \in R\}$.

$\operatorname{Im} L = \{y \in Z : Q_1 y = Q_2 y = 0\}$.

$y \in Z$, $x(t) = \frac{1}{2}\int_0^t (t-s)^2 y(s)ds + c_0 + c_1 t + c_2 t^2, c_0, c_1, c_2 \in R,$

$$\int_0^1 (1-s)y(s)ds - \sum_{i=1}^{m} \int_0^{\varepsilon_i} (\varepsilon_i - s)y(s)ds = 0, \tag{2}$$

$$\int_0^1 (1-s)^2 y(s)ds - \int_0^1 A(t)\int_0^t (t-s)^2 y(s)dsdt = 0, \qquad (3)$$

At the same time, it satisfies (1.2), so

$x \in domL$, $x''' \in \mathrm{Im}(L)$ $x(t) = \frac{1}{2}\int_0^t (t-s)^2 y(s)ds + c_0 + c_1 t + c_2 t^2, c_0, c_1, c_2 \in R,$

The conditional substitution expression can be satisfied by (2) and (3). Therefore $\mathrm{Im}\, L = \{y \in Z : Q_1 y = Q_2 y = 0\}$.

Construct a continuous linear map using the above definition $T_i(i=1,2) : Z \to Z,$

$$T_1 y(t) = \frac{12}{M}(M_{11}Q_1 y + M_{12}Q_2 y),$$

$$T_2 y(t) = \frac{60}{M}(M_{21}Q_1 y + M_{22}Q_2 y),$$

Defining continuous linear operators $Q : Z \to Z,$

$$Qy = (T_1 y(t))t + (T_1 y(t))t^2,$$

$$T_1((T_1 y)t) = \frac{12}{M}[M_{11}Q_1((T_1 y)t) + M_{12}Q_2((T_1 y)t)]$$

$$= \frac{12}{M}[M_{11}\frac{1}{6}(1-\sum_{i=1}^m a_i\varepsilon_i^3) + M_{12}\frac{1}{12}(1-\sum_{i=1}^m a_i\varepsilon_i^4)](T_1 y)$$

$$= \frac{[M_{11}2(1-\sum_{i=1}^m a_i\varepsilon_i^3) + M_{12}(1-\sum_{i=1}^m a_i\varepsilon_i^4)]}{M}(T_1 y)$$

$$= \frac{(T_1 y)}{M}[M_{11}m_{11} + M_{12}m_{12}]$$

$$= T_1 y,$$

$$T_1((T_2 y)t^2) = 0,\ T_2((T_1 y)t) = 0,\ T_2((T_2 y)t^2) = T_2 y.$$

Then $Q^2 y = Q(Qy) = T_1[(T_1 y(t))t + (T_1 y(t))t^2]t + T_2[(T_1 y(t))t + (T_1 y(t))t^2]t^2$

$= (T_1 y(t))t + (T_1 y(t))t^2 = Qy,$

Q is an idempotent operator. So Q is a linear projection operator.

If $Qy = 0,$

$$\begin{cases} M_{11}Q_1(y) + M_{12}Q_2(y) = 0, \\ M_{21}Q_1(y) + M_{22}Q_2(y) = 0. \end{cases}$$

For $\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix}\begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix} = \begin{vmatrix} M & 0 \\ 0 & M \end{vmatrix} = M^2 \neq 0,$ then $\begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix} \neq 0.$

Then $Q_1 y = Q_2 y = 0.$

$KerL = \mathrm{Im}(Q),\quad Z = \mathrm{Im}(Q) \oplus Ker(Q),$ then $Z = \mathrm{Im}\, L \oplus \mathrm{Im}\, Q.$

$\dim KerL = co\dim \mathrm{Im}\, L = 3,$ $L : d\quad o \cap Ym \to ZL$. Is a zero indicator operator *Fredholm* .

$P : X \to X,$

$$Px(t) = x(0) + x'(0)t + \frac{1}{2}x''(0)t^2, t \in [0,1],$$

Evidence P is a continuous projection operator, and $KerP = \{x : x(0) = x'(0) = x''(0) = 0\},$

$X = KerL \oplus KerP.$

Redefinition operator $K_p: \mathrm{Im} L \to dom L \cap Ker P$, $(K_P y)(t)=\frac{1}{2}\int_0^t (t-s)^2 y(s)ds.$

For $y\in \mathrm{Im}\, L$, $(LK_P)y(t)=(K_P y)'''=y(t).$

for $x\in dom(L)\cap KerP,$

$$(K_P L)x(t)=\frac{1}{2}\int_0^t (t-s)^2 x'''(s)ds$$

$$=x(t)-(x(0)+x'(0)t+\frac{1}{2}x''(0)t^2)=x(t),$$

For $(K_P L)x(t)=x(t)$. This proves that $K_p=(L|_{domL\cap KerP})^{-1}$.

and $\|K_P y\|_\infty \le \|y\|_1, \|(K_P y)'\|_\infty \le \|y\|_1, \|(K_P y)''\|_\infty \le \|y\|_1$,

So $\|K_P y\|=\|K_P y\|_\infty+\|(K_P y)'\|_\infty+\|(K_P y)''\|_\infty \le 3\|y\|_1$.

Lemma 3.2 $K_p(I-Q)N: X\to X$ Full continuous.

The proof of this lemma is similar to [5], which is omitted here.Theorem 3.1 assumes that the function $f:[0,1]\times R^3\to R$ satisfies the Caratheodory condition, and

$(H1)$ is for random $(x,y,z)\in R^3$ and all $t\in[0,1]$,has a function $a(t), b(t), c(t), d(t), m(t),$ $n(t)\in L_1[0,1]$ and parameter $\theta,\varphi\in[0,1]$,Make one of the following inequalities true. $L(t)=a(t)+b(t)|x|+c(t)|y|+d(t)|z|,$

$$|f(t,x,y,z)|\le L(t)+m(t)|y|^\theta+n(t)|z|^\varphi, \tag{4}$$

$$|f(t,x,y,z)|\le L(t)+m(t)|z|^\theta+n(t)|y|^\varphi. \tag{5}$$

$(H2)$ has a parameter $A>0$, Make it all $t\in[0,1], x\in domL\setminus KerL$,

If $|x|+|x'|+|x''|>A$,so $Q_1 Nx\ne 0,$ or $Q_2 Nx\ne 0.$

$(H3)$ has a parameter $B>0$,and make random $a,b\in R$,if $a^2+b^2>B,$ so

$$aT_1 N(at+bt^2)+bT_2 N(at+bt^2)>0, \tag{6}$$

$$aT_1 N(at+bt^2)+bT_2 N(at+bt^2)<0. \tag{7}$$

$\|b\|_1+\|c\|_1+\|d\|_1<\frac{1}{9}$, then the resonance boundary value problem (1.1)-(1.2) has at least one solution.

Proof: (1) Order $\Omega_1=\{x\in dom(L)\setminus KerL: Lx=\lambda Nx, \lambda\in[0,1]\}$,

$x\in\Omega_1, x\notin KerL$,then $Lx=\lambda Nx$, $\lambda\ne 0,$ $Nx\in \mathrm{Im}\, L=KerQ,$

$QNx=0,$ $Q_1 Nx=Q_1 Nx=0.$ $(H2)$, $t_0\in[0,1]$,

$|x(t_0)|+|x'(t_0)|+|x''(t_0)|\le A.$ $x''(t)=x''(t_0)+\int_{t_0}^t x'''(s)ds,$

$$x'(t)=x'(t_0)+\int_{t_0}^t x''(s)ds,$$

$$x(t)=x(t_0)+\int_{t_0}^t x'(s)ds.$$

so

$$\left|x''(0)\right| \le \left\|x''(t)\right\|_\infty \le \left|x''(t_0)\right| + \left\|x'''(t)\right\|_1 \le \left|x''(t_0)\right| + \left\|Lx\right\|_1 \le A + \left\|Nx\right\|_1, \quad (8)$$

$$\left|x'(0)\right| \le \left\|x'(t)\right\|_\infty \le \left|x'(t_0)\right| + \left|x''(t_0)\right| + \left\|x'''(t)\right\|_1 \le A + \left\|Lx\right\|_1 \le A + \left\|Nx\right\|_1, \quad (9)$$

$$\left|x(0)\right| \le \left\|x(t)\right\|_\infty \le \left|x(t_0)\right| + \left|x'(t_0)\right| + \left|x''(t_0)\right| + \left\|x'''(t)\right\|_1 \le A + \left\|Lx\right\|_1 \le A + \left\|Nx\right\|_1, \quad (10)$$

Available in (8), (9), (10) $\left\|Px\right\| = \left\|x(0) + x'(0)t + \frac{1}{2}x''(0)t^2\right\|$

$$= \left\|x(0) + x'(0)t + \frac{1}{2}x''(0)t^2\right\|_\infty + \left\|x'(0) + x''(0)t\right\|_\infty + \left\|x''(0)\right\|_\infty$$

$$\le \left\|x(0)\right\|_\infty + 2\left\|x'(0)\right\|_\infty + 3\left\|x''(0)\right\|_\infty$$

$$\le 6A + 6\left\|Nx\right\|_1. \quad (11)$$

$x \in \Omega_1$, $(I-P)x \in domL \cap KerL$, $LPx = 0$. then

$$\left\|(I-P)x\right\| = \left\|K_P L(I-P)x\right\| \le 3\left\|L(I-P)x\right\|_1 \le 3\left\|Lx\right\|_1 \le 3\left\|Nx\right\|_1. \quad (12)$$

Comprehensive (11), (12) can be obtained

$$\left\|x\right\| = \left\|Px + (I-P)x\right\| \le \left\|Px\right\| + \left\|(I-P)x\right\| \le 6A + 9\left\|Nx\right\|_1. \quad (13)$$

If (4) is established, it is available from (13)

$$\left\|x\right\| \le 9(\left\|a\right\|_1 + \left\|b\right\|_1\left\|x\right\|_\infty + \left\|c\right\|_1\left\|x'\right\|_\infty + \left\|d\right\|_1\left\|x''\right\|_\infty + \left\|m\right\|_1\left\|x'\right\|_\infty^\theta + \left\|n\right\|_1\left\|x''\right\|_\infty^\varphi + \left\|e\right\|_1) + 6A. \quad (14)$$

Calculated by calculation:

$$\text{n}\left\|x\right\|_\infty \le \frac{9(\left\|a\right\|_1 + \left\|c\right\|_1\left\|x'\right\|_\infty + \left\|d\right\|_1\left\|x''\right\|_\infty + \left\|m\right\|_1\left\|x'\right\|_\infty^\theta + \left\|n\right\|_1\left\|x''\right\|_\infty^\varphi + \left\|e\right\|_1) + 6A}{1 - 9\left\|b\right\|_1}, \quad (15)$$

$$\left\|x'\right\|_\infty \le \frac{9(\left\|a\right\|_1 + \left\|d\right\|_1\left\|x''\right\|_\infty + \left\|n\right\|_1\left\|x''\right\|_\infty^\varphi + \left\|e\right\|_1) + (\left\|m\right\|_1\left\|x'\right\|_\infty^\theta + 6A)}{1 - 9\left\|b\right\|_1 - 9\left\|c\right\|_1}, \quad (16)$$

$$\left\|x''\right\|_\infty \le \frac{9(\left\|a\right\|_1 + \left\|m\right\|_1\left\|x'\right\|_\infty^\theta + \left\|e\right\|_1) + (9\left\|n\right\|_1\left\|x''\right\|_\infty^\varphi + 6A)}{1 - 9\left\|b\right\|_1 - 9\left\|c\right\|_1 - 9\left\|d\right\|_1}. \quad (17)$$

$\theta, \varphi \in [0,1]$ $\left\|b\right\|_1 + \left\|c\right\|_1 + \left\|d\right\|_1 < \frac{1}{9}$ When established, there are constants at the same time $M_1, M_2, M_3 > 0$, Make it all $x \in \Omega_1$ $\left\|x''\right\|_\infty \le M_1, \left\|x'\right\|_\infty \le M_2, \left\|x\right\|_\infty \le M_3$.

Available from (14)- (17) $\left\|x\right\| = \left\|x\right\|_\infty + \left\|x'\right\|_\infty + \left\|x''\right\|_\infty \le M_1 + M_2 + M_3$.

Therefore, it is bounded under the conditions of (4). If (5) is established, the same reason can be bounded.

(2) Order $\Omega_2 = \{x \in KerL : Nx \in \operatorname{Im} L\}$

$x \in \Omega_2$, $x \in KerL = \{x \mid x = at + bt^2, a, b \in R\}$,

$Nx \in \operatorname{Im} L$ and $\operatorname{Im} L = KerQ$, $QNx = 0$, $T_1 Nx(t) = T_2 Nx(t) = 0$.

so $(H3)$ $a^2 + b^2 \le B$,so Ω_2 has a boundary

(3) Assumption (6) holds. Define linear isomorphic mapping $J : KerL \to \operatorname{Im} Q$,

$$J(at + bt^2) = \frac{12}{M}(aM_{11} + bM_{12})t + \frac{60}{M}(aM_{21} + bM_{21})t^2.$$

$\Omega_3=\{x\in KerL: \lambda Jx+(1-\lambda)QNx=0, \lambda\in[0,1]\}$

For random $x=at+bt^2\in\Omega_3$,so $\lambda Jx+(1-\lambda)QNx=0$,then

$$\begin{cases}\frac{12M_{11}}{M}[\lambda a+(1-\lambda)T_1N(at+bt^2)]+\frac{12M_{12}}{M}[\lambda b+(1-\lambda)T_2N(at+bt^2)]=0,\\ \frac{60M_{21}}{M}[\lambda a+(1-\lambda)T_1N(at+bt^2)]+\frac{60M_{22}}{M}[\lambda b+(1-\lambda)T_2N(at+bt^2)]=0.\end{cases}$$

It is $\begin{cases}M_{11}[\lambda a+(1-\lambda)T_1N(at+bt^2)]+M_{12}[\lambda b+(1-\lambda)T_2N(at+bt^2)]=0,\\ M_{21}[\lambda a+(1-\lambda)T_1N(at+bt^2)]+M_{22}[\lambda b+(1-\lambda)T_2N(at+bt^2)]=0.\end{cases}$

Because $\begin{vmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{vmatrix}\neq 0,$ $\begin{cases}\lambda a+(1-\lambda)T_1N(at+bt^2)=0,\\ \lambda b+(1-\lambda)T_2N(at+bt^2)=0.\end{cases}$

if $\lambda=1$, $a=b=0.$

if $\lambda\in[0,1)$,then $\lambda(a^2+b^2)=-(1-\lambda)[aT_1N(at+bt^2)+bT_2N(at+bt^2)]<0$,

$\lambda(a^2+b^2)\geq 0$,then Ω_3 has its boundary.

If (7) is established, $\Omega_3=\{x\in KerL: -\lambda Jx+(1-\lambda)QNx=0, \lambda\in[0,1]\}$ Ω_3 has its boundary

(4) Take the bounded set $\Omega:\overline{\Omega_i}\subset\Omega$:

1) $Lx\neq\lambda Nx$, $(x,\lambda)\in[(domL\setminus KerL)\cap\partial\Omega]\times(0,1)$;

2) $Nx\notin \operatorname{Im} L$, $x\in KerL\cap\partial\Omega$;

The formula (3) of the testimony theorem A is established.

Construct a homotopy equation: $H(x,\lambda)=\pm\lambda Jx+(1-\lambda)QNx,$ Known by the same nature:

$$\deg(H(QN|_{KerL}, KerL\cap\Omega,0)=\deg(H(\cdot,0),KerL\cap\Omega,0)$$
$$=\deg(H(\cdot,1),KerL\cap\Omega,(0,0))=\deg(\pm I,KerL\cap\Omega,(0,0))$$
$$=\operatorname{sgn}(\pm\begin{vmatrix}\frac{12M_{11}}{M} & \frac{12M_{12}}{M}\\ \frac{60M_{21}}{M} & \frac{60M_{21}}{M}\end{vmatrix})$$
$$=\operatorname{sgn}(\pm 720\begin{vmatrix}M_{11} & M_{12}\\ M_{21} & M_{21}\end{vmatrix})$$
$$\neq 0.$$

Therefore, if the condition (3) of the theorem A holds, then the operator equation has at least one solution.

Therefore, it can be seen that the resonance boundary value problem (1.1)-(1.2) has at least one solution.

Theorem 3.2 The hypothesis function satisfies the Caratheodory condition, satisfies the sum of theorem 3.1, and has the following decomposition: $f(t,x,y,z)=g(t,x,y,z)+h(t,x,y,z)$;

$(H4)$ for a random $(t,x,y,z)\in[0,1]\times R^3$, $z[g(t,x,y,z)+e]<0$;

$(H5)$ $a,b,c,d,m,n,p\in L^1[0,1]$,it makesfor a random $(x,y,z)\in R^3$,parameter θ_1,θ_2,

$\theta_3\in[0,1]$,it makes

$|h(t,x,y,z)|\leq a(t)+b(t)|x|+c(t)|y|+d(t)|z|+m|x|^{\theta_1}+n|y|^{\theta_2}+p|z|^{\theta_3}$;

if the function $f(t,x,y,z)$ The above conditions are satisfied, and at least one solution exists for the resonance boundary value problem (1.1)-(1.2).

To prove:(1) $\Omega_1=\{x\in dom(L)\setminus KerL: Lx=\lambda Nx, \lambda\in[0,1]\}$

for a random $x\in\Omega_1$, $x\notin KerL$,then $Lx=\lambda Nx$, $\lambda\neq 0,$ $Nx\in \operatorname{Im} L=KerQ,$

so $QNx=0,\quad Q_1Nx=Q_1Nx=0.\quad (H2)$, $t_0\in[0,1]$,it makes $|x(t_0)|+|x'(t_0)|+|x''(t_0)|\le A.$

Furthermore, the nature of the differential is known:

$$x'(t)=x'(t_0)+\int_{t_0}^{t}x''(s)ds, \tag{18}$$

$$x(t)=x(t_0)+\int_{t_0}^{t}x'(s)ds. \tag{19}$$

then:

$$\|x'\|_\infty\le|x(t_0)|+\|x''\|_\infty\le A+\|x''\|_\infty, \tag{20}$$

$$\|x\|_\infty\le|x(t_0)|+|x'(t_0)|+\|x''\|_\infty\le A+\|x''\|_\infty. \tag{21}$$

$x''(t)x'''(t)=\lambda x''(t)[f(t,x(t),x'(t),x''(t))+e(t)]$

Integrate both sides at the same time, there is

$$\frac{1}{2}(x''(t))^2$$

$$=\frac{1}{2}(x''(t_0))^2+\lambda\int_{t_0}^{t}x''(s)[g(s,x(s),x'(s),x''(s))+e(s)]ds+\lambda\int_{t_0}^{t}x''(s)[h(s,x(s),x'(s),x''(s))+e(s)]ds$$

$$\le\frac{A^2}{2}+\int_0^1|x''(s)||[h(s,x(s),x'(s),x''(s))+e(s)]|ds$$

$$\le\frac{A^2}{2}+\|x''\|_\infty[\|a\|_1+\|b\|_1\|x\|_\infty+\|c\|_1\|x'\|_\infty+\|d\|_1\|x''\|_\infty+\|m\|_1\|x\|_\infty^{\theta_1}+\|n\|_1\|x'\|_\infty^{\theta_2}+\|p\|_1\|x''\|_\infty^{\theta_3}+\|e\|_1]$$

$$\le\frac{A^2}{2}\|x''\|_\infty[\|a\|_1+\|b\|_1A+\|b\|_1\|x''\|_\infty+\|c\|_1A+\|c\|_1\|x''\|_\infty+\|d\|_1\|x''\|_\infty$$

$$+\|m\|_1\|x\|_\infty^{\theta_1}+\|n\|_1\|x'\|_\infty^{\theta_2}+\|p\|_1\|x''\|_\infty^{\theta_3}+\|e\|_1]$$

$$\le\frac{A^2}{2}+(\|x''\|_\infty)^2[\|b\|_1+\|c\|_1+\|d\|_1]$$

$$+\|x''\|_\infty[\|a\|_1+\|b\|_1A+\|c\|_1A++\|m\|_1\|x\|_\infty^{\theta_1}+\|n\|_1\|x'\|_\infty^{\theta_2}+\|p\|_1\|x''\|_\infty^{\theta_3}+\|e\|_1]$$

Then

$$(\|x''\|_\infty)^2(\frac{1}{2}-\|b\|_1-\|c\|_1-\|d\|_1)\le\|x''\|_\infty[\|a\|_1+\|b\|_1A+\|c\|_1A+\|m\|_1\|x\|_\infty^{\theta_1}+\|n\|_1\|x'\|_\infty^{\theta_2}+\|p\|_1\|x''\|_\infty^{\theta_3}+\|e\|_1]+\frac{A^2}{2}$$

$$(\|x''\|_\infty)^2\le\frac{\|x''\|_\infty[\|a\|_1+\|b\|_1A+\|c\|_1A+\|m\|_1\|x\|_\infty^{\theta_1}+\|n\|_1\|x'\|_\infty^{\theta_2}+\|p\|_1\|x''\|_\infty^{\theta_3}+\|e\|_1]+\frac{A^2}{2}}{\frac{1}{2}-\|b\|_1-\|c\|_1-\|d\|_1}.$$

For $\theta_1,\theta_2,\ \theta_3\in[0,1]$ and $\|b\|_1+\|c\|_1+\|d\|_1<\frac{1}{2}$, parameter

parameter $M_1>0$ it makes $\|x''\|_\infty\le M_1$.

(20),(21)can get $\|x'\|_\infty\le A+M_1, \|x\|_\infty\le A+M_1$.

so $\|x\|=\|x\|_\infty+\|x'\|_\infty+\|x''\|_\infty\le 2A+3M_1$.

so Ω_1 It is bounded. The remaining proof is exactly the same as the proof in the second half of

Theorem 3.1, which is omitted here.

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References

[1] Liu Xiaogang, Ouyang Zigen, Hui Xiaojian. Boundary Value Problems for Fractional Differential Equations with P-Laplacian Operators under Resonant Conditions [J]. Journal of Natural Science of Xiangtan University, 2018, 40(1), 44- 47.

[2] Sun Qian, Liu Wenbin. Existence of Positive Solutions for a Class of Singular Fractional Differential Equations with Integral Boundary Value Problems[J].Mathematics in Practice and Theory,2017,47(17):295-306.

[3] Liu Xiaogang, Zhong Jichao, Ren Ruichao, Hui Xiaojian, Ouyang Zigen. Existence of Solutions for Boundary Value Problems of Fractional Coupled Systems with Resonant Conditions [J].Mathematics in Practice and Theory, 2018, 48(10) :216-224.

[4], Wu Yuhan, Li Chengyue. Multiple Existence of Solutions for Resonance Problems of Fourth Order Differential Equations [J], Journal of Applied Functional Analysis, 2016, 18(2): 141-149.

[3] Du Ruijuan. Existence of solutions for second-order multi-point boundary value problems under resonance [J], Mathematics Practice and Cognition, 2015.45(24): 272-278.

[4] Li Yuhua, Liang Zhangping. Two positive solutions to three-point singular boundary value problem value problems. Acta Math Sci, 2011, 31B (1): 29-38.

[5] Liu Xiaogang, Ouyang Zigen. Existence of Solutions for a Class of Fractional Multi-Point Boundary Value Problems with Resonance Conditions[J], Journal of Natural Science of Xiangtan University, 2017, 39(2): 121-124.

[7] Zhang Di,Liu Wenbin. Existence of solutions for resonance infinite multi-point boundary value problems of fractional differential equations with P-laplace operator[J].Journal of Shandong University, Journal of Science,2017,52(17):72 -80.

[5] Chen Fulai. Coincidence degree and fractional differential equntions with impulses [J]. Computers and Mathematics with Applications, 2012, 64: 3444-3455.

[8] Hu Lei, Zhang Shuqin, Shi Ai-ling. Existence of Solutions for Resonance Boundary Value Problems of Coupled Systems of Fractional Differential Equations[J].Journal of Mathematical Physics, 2014, 34A(5): 1313-1326.

[9] Ge Weigao. Boundary Value Problems for Nonlinear Differential Equations [M]. Beijing: Science and Technology Press, 2009.

[10] Guo Dazhao. Nonlinear Functional Analysis (Second Edition). Jinan: Shandong Science and Technology Press, 2003.